LIMITS OF APPLICABILITY OF QUASISTATIONARY VALUES OF HEAT-TRANSFER COEFFICIENTS IN CALCULATING REAL NONSTATIONARY THERMAL PROCESSES

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Experimental evidence is presented for the limits of applicability of quasistationary relations in calculating actual nonstationary thermal processes involving the flow of gases and liquids in tubes.

It was shown in [1, 2] and in a number of other papers that under nonstationary conditions the heat-transfer coefficient in channels is appreciably different from quasistationary values. It was shown in [2] that appreciable errors may result from calculating nonstationary thermal processes with quasistationary values of the heat-transfer coefficient.

For the turbulent flow of coolants the difference between the nonstationary and quasistationary values of the heat-transfer coefficient increases with increasing absolute values of the thermal and hydrodynamic instability parameters or the first time derivatives of the wall temperature or the heat flux density at the wall and the flow rate. Since these derivatives are not generally known beforehand, actual calculations of nonstationary thermal processes must be performed by the method of successive approximation.

Thus, taking account of the effect of unsteadiness on the heat-transfer coefficient complicates the calculation, and therefore from the practical point of view it is useful to know in which cases it is possible to neglect this effect and to use quasistationary values of the heat-transfer coefficients.

The generalized relations given in [1-3] for the nonstationary heat-transfer coefficient for the flow of gases and liquids in tubes permit the determination of the limits of applicability of the quasistationary method of calculation in each specific problem. In order to do this it is necessary to specify the error in the determination of the heat-transfer coefficient

$$\Delta K = K - 1, \tag{1}$$

which is admissible for the given problem, and for a given value of K it is possible to find limiting values of the thermal or hydrodynamic instability parameters and corresponding values of the time derivatives of the wall temperature $\partial T_w/\partial\tau$ or the flow rate $\partial G/\partial\tau$ for given values of the other parameters (for gases Re_b, T_w/T_b; for liquids Re_b, Pr_b).

For the flow of gases in tubes and time rate of change of the wall temperature or the heat flux density at the wall for a constant mass flow rate, the generalized relations for the nonstationary heat-transfer coefficient have the form

$$K = f(K^*_{Tg}, \mathrm{Re}_b, T_w/T_b). \tag{2}$$

The thermal instability parameter $K^*_{Tg} = (\partial T_w/\partial\tau)\beta_w a \sqrt{(\lambda_b/c_{pb} gG)}$ takes account of the effect of the change of structure of turbulent flow on nonstationary heat transfer.

Equations (2) given in [1, 2] give the following limiting values of the thermal instability parameter for the heating of gases in tubes.

1. For an increase in heat flux density or wall temperature ($\Delta K > 0$)

$$K^*_{Tg} = \left[\frac{\Delta K}{\left(2 - 0.83\frac{T_w}{T_b}\right)(10.4 - 19.2\,\mathrm{Re}_b \cdot 10^{-5})}\right]^{\frac{1}{1.836 - 0.664\,\mathrm{Re}_b \cdot 10^{-5}}} \cdot 10^{-4} \tag{3}$$

for $\mathrm{Re}_b = 7 \cdot 10^3 - 2.5 \cdot 10^4$; $T_w/T_b = 1 - 1.7$;

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$$K_{Tg}^{*}=\left[\frac{\Delta K}{\left(2-0.83\frac{T_w}{T_b}\right)(4.6-1.46\,\mathrm{Re}_b\cdot 10^{-5})}\right]^{\frac{1}{1.605-0.1\,\mathrm{Re}_b\cdot 10^{-5}}}\cdot 10^{-4} \quad (4)$$

for $\mathrm{Re}_b = 2.5\cdot 10^4 - 2\cdot 10^5$; $T_w/T_b = 1 - 1.7$;

2. For a decrease in heat flow density or wall temperature ($\Delta K < 0$)

$$K_{Tg}^{*}=-10^{-4}\left[\frac{1+\dfrac{\Delta K}{1.25\,(2-T_w/T_b)}}{0.325+0.206\,\mathrm{Re}_b\cdot 10^{-5}}\right]^{\frac{1}{0.105\,\mathrm{Re}_b\cdot 10^{-5}-0.27}} \quad (5)$$

for $\mathrm{Re}_b = 7\cdot 10^3 - 2\cdot 10^5$; $T_w/T_b = 1 - 1.7$; $|\Delta K| > 0.0625\,(2-T_w/T_b)\;(4.85-2.2\,\mathrm{Re}_b\cdot 10^{-5})$;

$$K_{Tg}^{*}=\frac{\Delta K}{1.25\left(2-\dfrac{T_w}{T_b}\right)(4.85-2.2\,\mathrm{Re}_b\cdot 10^{-5})}\cdot 10^{-4} \quad (6)$$

for $\mathrm{Re}_b = 7\cdot 10^3 - 2\cdot 10^5$; $T_w/T_b = 1 - 1.7$; $|\Delta K| < 0.0625\left(2-\dfrac{T_w}{T_b}\right)(4.85-2.2\,\mathrm{Re}_b\cdot 10^{-5})$;

$$K_{Tg}^{*}=0.217\cdot 10^{-4}\ln\left[1+\frac{\Delta K}{0.5\dfrac{T_w}{T_b}-0.42}\right] \quad (7)$$

for $\mathrm{Re}_b = 8\cdot 10^4 - 5.2\cdot 10^5$; $T_w/T_b = 1 - 1.6$.

For a given value of ΔK the limiting admissible values of the parameter K_{Tg}^{*} are larger the larger Re_b and T_w/T_b, i.e., the smaller the effect of thermal instability on the heat-transfer coefficient. Figure 1 shows the limiting values of K_{Tg}^{*} for which $|\Delta K| = 0.1$ and 0.05, i.e., the error in determining the nonstationary coefficient resulting from not taking account of the effect of thermal instability does not exceed 10 and 5%, respectively.

For the nonstationary cooling of a gas in tubes shown in Fig. (2) [1, 4] gives the following limiting value of the thermal instability parameter:

$$K_{Tg}^{*}=\frac{\Delta K}{C_0}\cdot 10^{-5}, \quad (8)$$

where

$$C_0=\left\{\left[14.97\left(\frac{T_w}{T_b}\right)^3-16.07\left(\frac{T_w}{T_b}\right)^2-0.526\frac{T_w}{T_b}+3.193\right]\times\right.$$
$$\left.\times(\mathrm{Re}_b\cdot 10^{-5})^{\left(1.25-3\frac{T_w}{T_b}\right)}+46.77\left(\frac{T_w}{T_b}\right)^3-119.1\left(\frac{T_w}{T_b}\right)^2+99.09\frac{T_w}{T_b}-27.08\right\},$$

$\mathrm{Re}_b = 3.2\cdot 10^4 - 2\cdot 10^5$; $T_w/T_b = 0.6 - 0.95$.

The limiting value of K_{Tg}^{*} is larger the larger Re_b and the smaller T_w/T_b, i.e., the smaller the effect of the thermal instability on K. Figure 2 shows the limiting values of K_{Tg}^{*} for which $\Delta K = 0.1$ and 0.05 for the cooling of a gas. As can be seen from Figs. 1 and

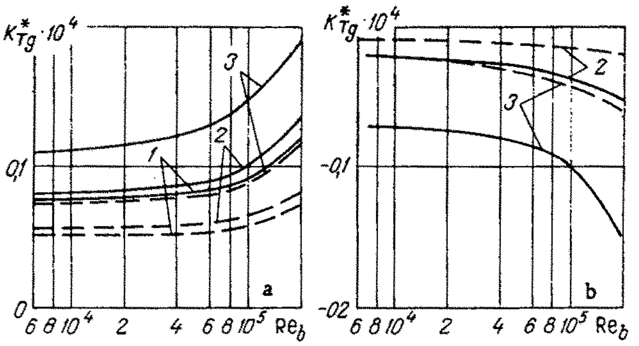

Fig. 1. Limiting values of the parameters K_{Tg}^{*} for which the error in using the quasistationary relations does not exceed 10% (solid curves) and 5% (open curves) for the heating of a gas in a tube for a) an increase and b) a decrease of the heat load: 1, 2, 3) T_w/T_b = 1, 1.2, 1.7.

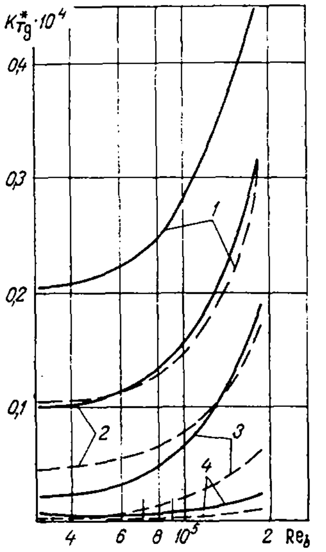

Fig. 2. Limiting values of the parameter K^*_{Tg} for which the error in using the quasistationary relations does not exceed 10% (solid curves) and 5% (open curves) for the cooling of a gas in tubes: 1, 2, 3, 4) T_w/T_b = 0.65, 0.75, 0.85, 0.95.

2 when the temperature factor $T_w/T_b \simeq 1$ for cooling at a given value of ΔK, the limiting value of K^*_{Tg} is smaller than for the heating of a gas (for $K^*_{Tg} > 0$), but close to the limiting value of $|K^*_{Tg}|$ for a decrease of the heat load ($K^*_{Tg} < 0$).

The relations presented enable us, in particular, to determine that in thermal power plant equipment designed for lower heating rates to limit thermal stresses, the heat-transfer coefficients, as a rule, differ only slightly from the quasistationary values.

For example, it follows from Eq. (8) that if the walls of steam pipes are heated at the rate $\partial T_w/\partial\tau$ = 40-60°K/h and d = 0.1-0.3 m, the parameter K^*_{Tg} does not exceed 10^{-9}-10^{-7}. The difference between the nonstationary and quasistationary heat-transfer coefficients in the range T_w/T_b = 0.65-0.95 does not exceed 2% for $Re_b = 2 \cdot 10^4$ and 0.01% for $Re_b = 2 \cdot 10^5$.

It was shown in [2, 3] that at a constant flow rate the relation for nonstationary heat transfer for the flow of liquids in tubes has the form

$$K = f(K_q,\ K^*_{Tg},\ \mathrm{Re}_b,\ \mathrm{Pr}_b) \tag{9}$$

or

$$K = 1 + \Delta K_1(K_q,\ \mathrm{Re}_b,\ \mathrm{Pr}_b) + \Delta K_2(K^*_{Tg},\ \mathrm{Re}_b), \tag{10}$$

where ΔK_1 takes account of the effect of variable thermal conductivity on the nonstationary heat-transfer coefficient and ΔK_2 describes the effect of the change in the turbulent structure of the flow: $K_q = (\partial q_w/\partial\tau)(d^2/q_w a_b)$.

The limiting values of the thermal instability parameters K_q and K^*_{Tg} are obtained by finding a relation between them.

Since $q_w = \alpha(T_w - T_b)$,

$$\frac{\partial q_w}{\partial\tau} = \frac{\partial T_w}{\partial\tau}\alpha - \frac{\partial T_b}{\partial\tau}\alpha + \frac{\partial\alpha}{\partial\tau}(T_w - T_b). \tag{11}$$

Since the last two terms in Eq. (11) are small in comparison with the first, we can write

$$\frac{\partial q_w}{\partial\tau} \simeq \frac{\partial T_w}{\partial\tau}\alpha. \tag{12}$$

Then

$$\frac{K_q}{K^*_{Tg}} = \frac{d\sqrt{c_{pb} g G}}{a_b (T_w - T_b)\beta_w \sqrt{\lambda_b}} = \sqrt{\frac{\pi}{4}\mathrm{Re}_b \mathrm{Pr}_b}\,\frac{d^{3/2} g^{1/2}}{(T_w - T_b)\beta_w a_b} \tag{13}$$

and

$$K^*_{Tg} = \frac{K_q}{\sqrt{\frac{\pi}{4}\mathrm{Re}_b \mathrm{Pr}_b}}\,C,$$

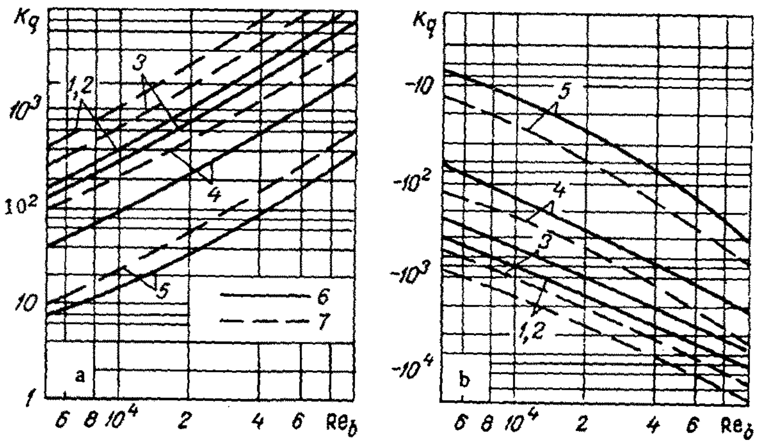

Fig. 3. Limiting values of the parameter K_q for which the error caused by using the quasistationary relations does not exceed 10% (a, b, increase and decrease of heat load) for the heating of a liquid in tubes: 1, 2, 3, 4, 5) $C = 10^{-9}$, 10^{-8}, 10^{-7}, 10^{-6}, 10^{-5}; 6, 7) Pr_b = 3, 10.

where

$$C = \frac{(T_w - T_b)\beta_w a_b}{d^{3/2} g^{1/2}}.$$

For water at T_b = 273-373°K, T_w = 280-373°K, $\beta_w = (0.7\text{-}8.1) \cdot 10^{-4}$/°K, $a_b = (1.33\text{-}1.69) \cdot 10^{-7}$ m²/sec, g = 9.8 m/sec², and d = 0.005-0.1 m, $C = 10^{-9}\text{-}10^{-5}$.

By using (13) Eqs. (10) presented in [2] for an increase in the heat flux density at the wall can be written in the form

$$\Delta K = \frac{26.6\, K_q^{0.71}}{\mathrm{Re}_b \mathrm{Pr}_q^{0.6}} + \frac{1.935 \cdot 10^6 K_q}{\mathrm{Re}_b^{0.803} \mathrm{Pr}_b^{0.5}} C \tag{14}$$

for $\mathrm{Re}_b = 5 \cdot 10^3 - 2 \cdot 10^4$; $\mathrm{Pr}_b = 3 - 10$;

$$\Delta K = \frac{26.6\, K_q^{0.71}}{\mathrm{Re}_b \mathrm{Pr}_b^{0.6}} + \frac{9.35 \cdot 10^9 K_q}{\mathrm{Re}_b^{1.66} \mathrm{Pr}_b^{0.5}} C \tag{15}$$

for $\mathrm{Re}_b = 2 \cdot 10^4 - 10^5$; $\mathrm{Pr}_b = 3 - 10$.

It can be seen from (14) and (15) that for given values of ΔK and C the quantities K_q and K_{Tg}^* are larger the larger Re_b and Pr_b, i.e., the smaller the effect of the thermal instability on K. Figure 3 shows solutions for Eqs. (14) and (15) for ΔK = 0.1 and 0.05, $C = 10^{-9}$, 10^{-8}, 10^{-7}, 10^{-6}, and 10^{-5}, $Re_b = 5 \cdot 10^3\text{-}10^5$, Pr_b = 3 and 10. The limiting values of K_q decrease with increasing C.

By using (13) and Eqs. (10) presented in [2] we obtain the limiting values of K_q for a decrease in be heat flux density at the wall (ΔK < 0):

$$K_q = \Delta K \left(\frac{2.4}{\mathrm{Re}_b \mathrm{Pr}_b^{0.6}} + \frac{1.935 \cdot 10^6}{\mathrm{Re}_b^{0.803} \mathrm{Pr}_b^{0.5}} C \right)^{-1} \tag{16}$$

for $\mathrm{Re}_b = 5 \cdot 10^3 - 2 \cdot 10^4$; $\mathrm{Pr}_b = 3 - 10$;

$$K_q = \Delta K \left(\frac{2.4}{\mathrm{Re}_b \mathrm{Pr}_b^{0.6}} + \frac{9.35 \cdot 10^9}{\mathrm{Re}_b^{1.66} \mathrm{Pr}_b^{0.5}} C \right)^{-1} \tag{17}$$

for $\mathrm{Re}_b = 2 \cdot 10^4 - 5 \cdot 10^4$; $\mathrm{Pr}_b = 3 - 10$. Figure 3 shows the limiting values of K_q for |ΔK| = 0.1 and various values of Re_b, Pr_b, and C. The absolute value of K_q is larger the larger Re_b and Pr_b and the smaller C.

Thus, for a given limiting value of the effect of thermal instability on the heat-transfer coefficient the above relations permit the determination of the thermal instability factors and the corresponding derivatives $\partial T_w/\partial\tau$ or $\partial q_w/\partial\tau$.

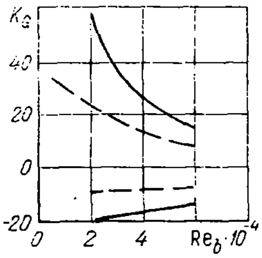

Fig. 4. Limiting values of the parameter K_G for which the error in using the quasistationary relation does not exceed 10% (solid curves) and 5% (open curves) for the heating of a liquid in a tube and the speeding up ($K_G > 0$) or slowing down ($K_G < 0$) of the flow.

The limits of applicability of the quasistationary relations can be determined in a similar way for a change in the flow rate of a coolant. If the admissible error in determining the heat-transfer coefficient $\Delta K_3 = K - 1$ resulting from the effect of hydrodynamic instability is specified, the limiting value of the parameter $K_G = (dG/d\tau)(d^2/G\nu)$ for the heating of a gas in a tube can be obtained from the relations given in [2]:

for slowing down of the flow

$$|K_G| = \left[\frac{1+\Delta K_3}{\left(0.66+0.275\frac{T_w}{T_b}\right)(0.915+0.08\,\mathrm{Re}_b\cdot 10^{-5})}\right]^{\frac{1}{0.25\,\mathrm{Re}_b\cdot 10^{-4}-0.16}}; \quad (18)$$

for speeding up of the flow

$$K_G = \left\{\frac{250\Delta K_3}{\left[\left(4.1-1.9\frac{T_w}{T_b}\right)+\frac{39.6-24.4\,T_w/T_b}{(\mathrm{Re}_b\cdot 10^{-4})^2}\right]}\right\}^{\frac{1}{2.4-1.4\,\mathrm{Re}_b\cdot 10^{-4}}} \quad (19)$$

Equation (18) is valid for $K_G = -30$ to -0.01, $Re_b = (1.5\text{–}8)\cdot 10^4$, $T_w/T_b = 1\text{–}1.7$. The limiting value of $|K_G|$ is larger the larger the admissible error in K and the larger Re_b and T_w/T_b, i.e., the smaller the effect of hydrodynamic instability on heat transfer. Equation (19) is valid for $K_G = 0\text{–}30$, $Re = (1\text{–}8)\cdot 10^4$, and $T_w/T_b = 1\text{–}1.7$.

Similar data for the limiting values of K_G can be obtained from the equations in [2] for the heating of a liquid in a tube. Figure 4 shows the limits of variation of K_G for ΔK_3 = 10 and 5% as functions of Re_b.

NOTATION

a, thermal diffusivity; c_p, specific heat; d, inside diameter of tube; G, mass flow rate; g = 9.8 m/sec^2; q_w, heat flux density at wall; T_w, wall temperature, T_b, mean mass flow temperature; α, heat-transfer coefficient; τ, time; β_w, coefficient of volume expansion of gas or liquid at wall temperature; λ, thermal conductivity; ν, kinematic viscosity; $K = Nu/Nu_0$; Nu, Nu_0, nonstationary and quasistationary values of Nusselt numbers; Re, Reynolds number; Pr, Prandtl number; K_q, K^*_{Tg}, thermal instability parameters; K_G, hydrodynamic instability parameter. Subscripts: w, at wall temperature; b, at mean mass flow temperature.

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